The period of 1/*k* for integer *k* is always < *k*

The meaning of the term 'period' is as follows. Take the case of 1/11 = 0.090909. Here the two digits 0 and 9 are repeated forever. So the period of 1/11 is 2. Another example is 1/7 = 0.142857142857… Here the 6 digits 142857 are repeated forever. So the period of 1/7 is 6. (There are lots of interesting properties for the number 1/7. For more details see this link)

Theorem: The period of 1/k for integer k is always < k

Proof:
The proof is indirect. Assume that the period of 1/*k* for integer *k* is more than k-1. We will see the proof with a specific example of 1/7. You can easily expand it to a general form. Assume the period of 1/7 is more than 6. Let it be 7 as below:

1/7 = 0.142857**3**142857**3**142… - (1)

Here the red colored **3** is the additional number we inserted purposefully.

We know that when we multiply 1/7 with an integer the result will be in the form $x\frac{y}{7}$ where x is an integer and y is another integer 0 ≤ y ≤ 6. Here x is the integer part and y/7 is the decimal part. Note that when we multiply 1/7 with any integer, the decimal part of the result will be in the form y/7 where y has 7 possibilities 0, 1, 2, 3, 4, 5 and 6. - (2)

From equation (1) we get:

10^0 X 1/7 = 0.14285731428573…
10^1 X 1/7 = 1.4285731428573…
10^2 X 1/7 = 14.285731428573…
10^3 X 1/7 = 142.85731428573…
10^4 X 1/7 = 1428.5731428573…
10^5 X 1/7 = 14285.731428573…
10^6 X 1/7 = 142857.31428573…

In the above 7 equations all the decimal part are different and none of the decimal part is 0. As per the statement (2) the decimal part is in the form y/7 but since the decimal part is non-zero y has only 6 possibilities 1, 2, 3, 4, 5 and 6. But in the above equations we got 7 different decimal parts which is a contradiction and hence the proof.